****

**CCGPS**

**Frameworks**

**Teacher Edition**

**Mathematics**

Unit 1: Relationships Between Quantities



*These materials are for nonprofit educational purposes only. Any other use may constitute copyright infringement.*

**Unit 1**

**Relationships Between Quantities**

Table of Contents

[OVERVIEW 3](#_Toc321124361)

[KEY STANDARDS 4](#_Toc321124362)

[ENDURING UNDERSTANDINGS 8](#_Toc321124363)

[ESSENTIAL QUESTIONS 8](#_Toc321124364)

[CONCEPTS/SKILLS TO MAINTAIN 9](#_Toc321124365)

[SELECTED TERMS AND SYMBOLS 10](#_Toc321124366)

[CLASSROOM ROUTINES 11](#_Toc321124367)

[STRATEGIES FOR TEACHING AND LEARNING 11](#_Toc321124368)

[EVIDENCE OF LEARNING 12](#_Toc321124369)

[TASKS 12](#_Toc321124370)

[Acting Out 14](#_Toc321124371)

[Lucy’s Linear Equations and Inequalities 20](#_Toc321124374)

[Forget the Formula 27](#_Toc321124376)

[Cara’s Candles Revisited 32](#_Toc321124378)

[The Yo-Yo Problem 37](#_Toc321124380)

[Paper Folding 43](#_Toc321124383)

[Culminating Task: Growing by Leaps and Bounds 49](#_Toc321124385)

# OVERVIEW

In this unit students will:

* interpret units in the context of the problem.
* convert units of measure using dimensional analysis
* when solving a multi-step problem, use units to evaluate the appropriateness of the solution.
* choose the appropriate units for a specific formula and interpret the meaning of the unit in that context.
* choose and interpret both the scale and the origin in graphs and data displays.
* determine and interpret appropriate quantities when using descriptive modeling.
* determine the accuracy of values based on their limitations in the context of the situation.
* identify the different parts of the expression and explain their meaning within the context of a problem.
* decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.
* create linear and exponential equations and inequalities in one variable and use them in a contextual situation to solve problems.
* create equations in two or more variables to represent relationships between quantities.
* graph equations in two variables on a coordinate plane and label the axes and scales.
* write and use a system of equations and/or inequalities to solve a real world problem.
* recognize that the equations and inequalities represent the constraints of the problem.
* solve multi-variable formulas or literal equations, for a specific variable.

The first unit of Coordinate Algebra involves relationships between quantities. Students will be provided with examples of real-world problems that can be modeled by writing an equation or inequality. The tasks begin with simple equations and inequalities and build up to equations in two or more variables. It is important to discuss using appropriate labels and scales on the axes when representing functions with graphs. Students will also explore examples illustrating when it is useful to rewrite a formula by solving for one of the variables in the formula.

In real-world situations, answers are usually represented by numbers associated with units. Units involve measurement and often require a conversion. Measurement involves both precision and accuracy. Estimation and approximation often precede more exact computations. Students need to develop sound mathematical reasoning skills and forms of argument to make reasonable judgments about their solutions. They should be able to decide whether a problem calls for an estimate, for an approximation, or for an exact answer. To accomplish this goal, teachers should provide students with a broad range of contextual problems that offer opportunities for performing operations with quantities involving units. These problems should be connected to science, engineering, economics, finance, medicine, or other career fields.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under “Evidence of Learning” be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

# KEY STANDARDS

**Reason quantitatively and use units to solve problems.**

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**Interpret the structure of expressions**

*Limit to linear expressions and to exponential expressions with integer exponents.*

**MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MCC9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**Create equations that describe numbers or relationships**

*Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas with a linear focus.*

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear ~~and quadratic functions, and simple rational~~ and exponential functions.

**MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations*.*

**Standards for Mathematical Practice**

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. **Make sense of problems and persevere in solving them.** High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
2. **Reason abstractly and quantitatively.** High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
3. **Construct viable arguments and critique the reasoning of others.** High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. **Model with mathematics.** High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
5. **Use appropriate tools strategically.** High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
6. **Attend to precision.** High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
7. **Look for and make use of structure.** By high school, students look closely to discern a pattern or structure. In the expression *x*2 + 9*x* + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(*x* – *y*)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.
8. **Look for and express regularity in repeated reasoning.** High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding (*x* – 1)(*x* + 1), (*x* – 1)(*x*2 + *x* + 1), and (*x* – 1)(*x*3 + *x*2 + *x* + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content**

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who do not have an understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a missing mathematical knowledge effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

# ENDURING UNDERSTANDINGS

* Identify the vocabulary for the parts that make up the whole expression. Interpret their meaning in terms of a context.
* Solve word problems where quantities are given in different units that must be converted to understand the problem.
* Select appropriate units for a specific formula and interpret the meaning of the unit in that context.
* Create linear and exponential equations and inequalities in one variable and use them in a contextual situation to solve problems.
* Recognize that exponential functions can be used to model situations of growth, including the growth of an investment through compound interest.
* Create equations in two or more variables to represent relationships between quantities.
* Graph equations in two variables on a coordinate plane and label the axes and scales.
* Write and use a system of equations and/or inequalities to solve a real world problem.
* Solve multi-variable formulas or literal equations for a specific variable in a linear expression.

# ESSENTIAL QUESTIONS

* How do I choose and interpret units consistently in formulas?
* How do I interpret parts of an expression in terms of context?
* How do I create equations and inequalities in one variable and use them to solve problems arising from linear and exponential functions?
* How can I write, interpret and manipulate algebraic expressions, equations, and inequalities?
* How do I create equations in two or more variables to represent relationships between quantities?
* How do I graph equations on coordinate axes with the correct labels and scales?
* How can I rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations?

# CONCEPTS/SKILLS TO MAINTAIN

Students may not realize the importance of unit conversion in conjunction with computation when solving problems involving measurement. Since today’s calculating devices often display 8 to 10 decimal places, students frequently express answers to a much greater degree of precision than is required.

Measuring commonly used objects and choosing proper units for measurement are part of the mathematics curriculum prior to high school. In high school, students experience a broader variety of units through real-world situations and modeling, along with the exploration of the different levels of accuracy and precision of the answers.

An introduction to the use of variable expressions and their meaning, as well as the use of variables and expressions in real-life situations, is included in the Expressions and Equations Domain of Grade 7.

Working with expressions and equations, including formulas, is an integral part of the curriculum in Grades 7 and 8. In high school, students explore in more depth the use of equations and inequalities to model real-world problems, including restricting domains and ranges to fit the problem’s context, as well as rewriting formulas for a variable of interest.

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess to determine whether instructional time should be spent on conceptual activities that help students develop a deeper understanding of these ideas.

* Using the Pythagorean Theorem
* Understanding slope as a rate of change of one quantity in relation to another quantity
* Interpreting a graph
* Creating a table of values
* Working with functions
* Writing a linear equation
* Using inverse operations to isolate variables and solve equations
* Maintaining order of operations
* Understanding notation for inequalities
* Being able to read and write inequality symbols
* Graphing equations and inequalities on the coordinate plane
* Understanding and using properties of exponents
* Graphing points
* Choosing appropriate scales and labeling a graph

# SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

**The definitions below are for teacher reference only and are not to be memorized by the students.** Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school children. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

<http://www.amathsdictionaryforkids.com/>

This web site has activities to help students more fully understand and retain new vocabulary.

<http://intermath.coe.uga.edu/dictnary/homepg.asp>

Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

* **Algebra:** The branch of mathematics that deals with relationships between numbers, utilizing letters and other symbols to represent specific sets of numbers, or to describe a pattern of relationships between numbers.
* **Coefficient:** A number multiplied by a variable.
* **Domain**:  The set of *x*-coordinates of the set of points on a graph; the set of *x*-coordinates of a given set of ordered pairs. The value that is the input in a function or relation.
* **Equation:** A number sentence that contains an equals symbol.
* **Expression:** A mathematical phrase involving at least one variable and sometimes numbers and operation symbols.
* **Function**:  A rule of matching elements of two sets of numbers in which an input value from the first set has only one output value in the second set.
* **Inequality**: Any mathematical sentence that contains the symbols > (greater than), < (less than), < (less than or equal to), or > (greater than or equal to).
* **Ordered Pair**:  A pair of numbers, (*x*, *y*), that indicate the position of a point on a Cartesian plane.
* **Perimeter**:  The sum of the lengths of the sides of a polygon.
* **Pythagorean Theorem:** It is a theorem that states a relationship that exists in any right triangle. If the lengths of the legs in the right triangle are *a* and *b* and the length of the hypotenuse is *c*, we can write the theorem as the following equation:

http://intermath.coe.uga.edu/dictnary/images/triangle/pythm.gif

* **Range**:  The *y*-coordinates of the set of points on a graph. Also, the *y*-coordinates of a given set of ordered pairs. The range is the output in a function or a relation.
* **Substitution:** To replace one element of a mathematical equation or expression with another.
* **Variable:** A letter or symbol used to represent a number.

# CLASSROOM ROUTINES

The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities as estimating, analyzing data, describing patterns, and answering daily questions. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, how to access classroom technology such as computers and calculators. An additional routine is to allow plenty of time for students to explore new materials before attempting any directed activity with these new materials. The regular use of routines is important to the development of students' number sense, flexibility, fluency, collaborative skills and communication. These routines contribute to a rich, hands-on standards-based classroom and will support students’ performances on the tasks in this unit and throughout the school year.

# STRATEGIES FOR TEACHING AND LEARNING

* Students should be actively engaged by developing their own understanding.
* Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols and words.
* Interdisciplinary and cross-curricular strategies should be used to reinforce and extend the learning activities.
* Appropriate manipulatives and technology should be used to enhance student learning.
* Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
* Students should write about the mathematical ideas and concepts they are learning.
* Consideration of all students should be made during the planning and instruction of this unit. Teachers need to consider the following:
  + What level of support do my struggling students need in order to be successful with this unit?
  + In what way can I deepen the understanding of those students who are competent in this unit?
  + What real life connections can I make that will help my students utilize the skills practiced in this unit?

# EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

* interpret the structure of expressions.
* create linear and exponential equations and inequalities in one variable and use them to solve problems.
* create equations in two or more variables to represent relationships.
* represent constraints by equations or inequalities, and by systems of equations and/or inequalities.
* interpret solutions in modeling context.
* rearrange linear formulas to highlight quantities of interest.

# TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all Coordinate Algebra students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task).

|  |  |
| --- | --- |
| **Scaffolding Task** | Tasks that build up to the learning task. |
| **Learning Task** | Constructing understanding through deep/rich contextualized problem solving tasks. |
| **Practice Task** | Tasks that provide students opportunities to practice skills and concepts. |
| **Performance Task** | Tasks which may be a formative or summative assessment that checks for student understanding/misunderstanding and or progress toward the standard/learning goals at different points during a unit of instruction. |
| **Culminating Task** | Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning. |
| **Formative Assessment Lesson (FAL)** | Lessons that support teachers in formative assessment which both reveal and develop students’ understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. |

|  |  |  |
| --- | --- | --- |
| **Task Name** | **Task Type**  ***Grouping Strategy*** | **Content Addressed** |
| Acting Out | Scaffolding Task  *Individual/Partner Task* | Model and write an equation in one variable Represent constraints with inequalities |
| Lucy’s Linear Equations and Inequalities | Practice Task  *Individual/Partner Task* | Write linear equations and inequalities in one variable and solve problems in context |
| Forget the Formula | Scaffolding Task  *Individual/Partner Task* | Creating equations in two variables to represent relationships  Represent constraints  Rearrange formulas to highlight a quantity of interest |
| Cara’s Candles Revisited | Scaffolding Task  *Individual/Partner Task* | Modeling equation to represent relationships  Determining constraints |
| The Yo-Yo Problem | Constructing Task  *Partner/Small Group Task* | Modeling linear patterns  Creating equation in one and two variables to represent relationships |
| Paper Folding | Constructing Task  *Partner/Small Group Task* | Modeling with exponential functions |
| Growing by Leaps and Bounds | Culminating Task  *Partner/Small Group Task* | Graph equations on coordinate axes with labels and scales  Interpret expressions that represent a quantity in terms of its context  Determining constraints  Modeling with exponential functions |

## Acting Out

**Mathematical Goals**

* Model and write an equation in one variable and solve a problem in context.
* Create one-variable linear equations and inequalities from contextual situations.
* Represent constraints with inequalities.
* Solve word problems where quantities are given in different units that must be converted to understand the problem.

**Common Core State Standards**

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational~~ and exponential functions.

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

**Introduction**

In this task, students will use an inequality to find the distance between two homes. Students will also use the Pythagorean Theorem to find the distance and learn how to convert contextual information into mathematical notation. The second part has students determine how much water might a dripping faucet waste in a year. Students will reason quantitatively and use units to solve problems.

**Materials**

* colored pencils
* compass (optional)

Part 1:

Erik and Kim are actors at a theater.

Erik lives 5 miles from the theater and Kim lives 3 miles from the theater.

Their boss, the director, wonders how far apart the actors live.

* On grid paper, pick a point to represent the location of the theater.
* Illustrate all of the possible places that Erik could live on the grid paper.
* Using a different color, illustrate all of the possible places that Kim could live on the grid paper.
* What is the smallest distance, *d*, that could separate their homes? How did you know?
* What is the largest distance, *d*, that could separate their homes? How did you know?
* Write and graph an inequality in terms of *d* to show their boss all of the possible distances that could separate the homes of the 2 actors.

***Comments***

***Students should understand that Erik and Kim could live anywhere on the circle with the theater as the center and the radius as the distance that they live from the theater.***

******

**2 mi**

**5 mi**

**3 mi**

***Solutions***

***Therefore, the closest that they could live would be 5 – 3 = 2 miles and the farthest apart that they could live would be 5 + 3 = 8 miles. This may be written as  where d represents the distance from Erik’s house to Kim’s house.***

***An inequality that could represent this distance could be 2 ≤ d ≤ 8 miles.***

***Graphing this inequality should look like the graph shown below.***

***• • • • • • • • • • • •***

***-1 0 1 2 3 4 5 6 7 8 9 10***

***Students should understand that the solid dots on the graph represent the fact that Erik and Kim could live exactly 2 miles or exactly 8 miles apart. Should the situation have been different and they lived more than 2 miles or less than 8 miles apart, those dots would have been left open, or not filled in. The space shaded on the number line between the 2 and 8 means that they could live any of those distances apart.***

Part 2:

The actors are good friends since they live close to each other. Kim has a leaky faucet in her kitchen and asks Erik to come over and take a look at it.

Kim estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds. Erik wants to know how many times the faucet drips in a week. Help Erik by showing your calculations below.

Kim estimates that approximately 575 drops fill a 100 milliliter bottle. Estimate how much water her leaky faucet wastes in a year.

***Solutions***

***60 sec = 1 min***

***60 min = 1 hour***

***24 hours = 1 day***

***7 days = 1 week***

***(60) (60) (24) (7) = 604800***

***604800/2 = 302,400 drops***

***365 days = 1 year***

***= 15,768,000 drops per year***

***15,768,000/575 = 27,422.608***

***About 27,423 of 100 millimeter bottles would be filled.***

## *27,423(100) = 2,742,300 milliliters* Acting Out

**Mathematical Goals**

* Model and write an equation in one variable and solve a problem in context.
* Create one-variable linear equations and inequalities from contextual situations.
* Represent constraints with inequalities.
* Solve word problems where quantities are given in different units that must be converted to understand the problem.

**Common Core State Standards**

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational~~ and exponential functions.

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

Part 1:

Erik and Kim are actors at a theater.

Erik lives 5 miles from the theater and Kim lives 3 miles from the theater.

Their boss, the director, wonders how far apart the actors live.

* On grid paper, pick a point to represent the location of the theater.
* Illustrate all of the possible places that Erik could live on the grid paper.
* Using a different color, illustrate all of the possible places that Kim could live on the grid paper.
* What is the smallest distance, *d*, that could separate their homes? How did you know?
* What is the largest distance, *d*, that could separate their homes? How did you know?
* Write and graph an inequality in terms of *d* to show their boss all of the possible distances that could separate the homes of the 2 actors.

Part 2:

The actors are good friends since they live close to each other. Kim has a leaky faucet in her kitchen and asks Erik to come over and take a look at it.

Kim estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds. Erik wants to know how many times the faucet drips in a week. Help Erik by showing your calculations below.

Kim estimates that approximately 575 drops fill a 100 milliliter bottle. Estimate how much water her leaky faucet wastes in a year.

## Lucy’s Linear Equations and Inequalities

**Mathematical Goals**

* Create one-variable linear equations and inequalities from contextual situations.
* Solve and interpret the solution to multi-step linear equations and inequalities in context.

**Common Core State Standards**

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational and~~ exponential functions.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**4. Model with mathematics.**

**7. Look for and make use of structure.**

**Introduction**

In this task, students will solve a series of linear equations and inequality word problems that Lucy has been assigned by her teacher. In order to help Lucy, students must explain in detail each step of the problem. It is a good idea to review key words that are associated with linear inequality word problems before beginning this task.

**Key words**:

fewer than \_\_\_\_ more than \_\_\_\_ at most \_\_\_\_

at least \_\_\_\_ less than \_\_\_\_ no less than \_\_\_\_

1. The sum of 38 and twice a number is 124. Find the number.

***Solution***

***Define a Variable: n = the number***

***Equation: 38 + 2n = 124***

***2n = 86***

***n = 43***

***The number is 43.***

***Check: 38 + 2(43) = 124***

***38 + 86 = 124***

***124 = 124***

1. The sum of two consecutive integers is less than 83. Find the pair of integers with the greatest sum.

***Solution***

***Define a Variable: x = the first consecutive number, x +1 = the second consecutive number***

***Equation: x + x + 1 < 83***

***2x < 82***

***x < 41***

***Answer: 40 and 41***

***Check: 40 + 41 < 83***

***81 < 83***

1. A rectangle is 12m longer than it is wide. Its perimeter is 68m. Find its length and width.

***Solution***

***Sketch:***

***w + 12***

***w w***

***w + 12***

***Define a Variable:***

***Width = w***

***Length = w + 12***

***Equation: w + w + w + 12 + w + 12 = 68***

***4w + 24 = 68***

***4w = 44***

***w = 11***

***Width = 11***

***Length = 11 + 12 = 23***

***So the width is 11m and the length is 23m.***

***Check: 11 + (11+12) + (11) + (11+12) = 68***

1. The length of a rectangle is 4 cm more than the width and the perimeter is at least 48 cm. What are the smallest possible dimensions for the rectangle?

***Solution***

***Sketch:***

***w + 4***

***w w***

***w + 4***

***Define a Variable:***

***Width = w***

***Length = w + 4***

***Equation: w + w + w + 4 + w + 4 ≥48***

***4w ≥ 40***

***w ≥ 10***

***Width = 10***

***Length = (10) + 4 = 14***

***So the width is 10cm and the length is 14cm.***

***Check: 10 + 10 + 14 + 14 ≥48***

***48 ≥ 48***

1. Find three consecutive integers whose sum is 171.

***Solution***

***Define a Variable:***

***x = the first consecutive number***

***x +1 = the second consecutive number***

***x + 2 = the third consecutive number***

***Equation: x + x + 1 + x + 2 = 171***

***3x + 3 = 171***

***3x = 168***

***x = 56***

***56 = the first consecutive number***

***56 +1 = the second consecutive number***

***56 + 2 = the third consecutive number***

***The three consecutive numbers are 56, 57, and 58.***

***Check: 56 + 57 + 58 = 171***

1. Find four consecutive even integers whose sum is 244.

***Solution***

***Define a Variable:***

***x = first number***

***x +2 = second number***

***x + 4 = third number***

***x + 6 = fourth number***

***Equation: x + x + 2 + x + 4 + x + 6 = 244***

***4x + 12 = 244***

***4x = 232***

***x = 58***

***58 = first number***

***58 +2 = second number***

***58 + 4 = third number***

***58 + 6 = fourth number***

***The numbers are 58, 60, 62, and 64.***

***Check: 58 + 60 + 62 + 64 = 244***

1. Alex has twice as much money as Jennifer. Jennifer has $6 less than Shannon. Together they have $54. How much money does each have?

***Solution***

***Define a Variable:***

***Shannon: s***

***Jennifer: s – 6***

***Alex: 2(s – 6)***

***Equation: s + s – 6 + 2(s – 6) = 54***

***2s – 6 + 2s – 12 = 54***

***4s – 18 = 54***

***4s = 72***

***s = 18***

***Shannon: s -> 18***

***Jennifer: s – 6 -> 12***

***Alex: 2(s – 6) -> 24***

***So, Shannon has $18, Jennifer has $12, and Alex has $24***

***Check: 18 + 12 + 24 = 54***

1. There are three exams in a marking period. A student received grades of 75 and 81 on the first two exams. What grade must the student earn on the last exam to get an average of no less than 80 for the marking period?

***Solution***

***Define a Variable:***

***x = last exam***

***Equation:***

***75 + 81 + x ≥ 240***

***x ≥ 84***

***The student must receive an 84 or higher exam grade in order to have an average no less than 80 for the marking period.***

***Check:***

## Lucy’s Linear Equations and Inequalities

**Mathematical Goals**

* Create one-variable linear equations and inequalities from contextual situations.
* Solve and interpret the solution to multi-step linear equations and inequalities in context.

**Common Core State Standards**

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational and~~ exponential functions.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**4. Model with mathematics.**

**7. Look for and make use of structure.**

Lucy has been assigned the following linear equations and inequality word problems. Help her solve each problem below by using a five step plan.

* Drawing a Sketch(if necessary)
* Defining a Variable
* Setting up an equation or inequality
* Solve the equation or inequality
* Make sure you answer the question

1. The sum of 38 and twice a number is 124. Find the number.
2. The sum of two consecutive integers is less than 83. Find the pair of integers with the greatest sum.
3. A rectangle is 12m longer than it is wide. Its perimeter is 68m. Find its length and width.
4. The length of a rectangle is 4 cm more than the width and the perimeter is at least 48 cm. What are the smallest possible dimensions for the rectangle?
5. Find three consecutive integers whose sum is 171.
6. Find four consecutive even integers whose sum is 244.
7. Alex has twice as much money as Jennifer. Jennifer has $6 less than Shannon. Together they have $54. How much money does each have?
8. There are three exams in a marking period. A student received grades of 75 and 81 on the first two exams. What grade must the student earn on the last exam to get an average of no less than 80 for the marking period?

## Forget the Formula

**Mathematical Goals**

* Rearrange formulas to highlight a quantity of interest.
* Create equations in two variables to represent relationships.
* Understand how the change in one variable affects the other variable in a given situation.
* Write and graph an equation to represent a linear relationship.
* Extend the concepts used in solving numerical equations to rearranging formulas for a particular variable.

**Common Core State Standards**

**MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations*.*

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**8. Look for and express regularity in repeated reasoning.**

**Introduction**

In this task, students will develop a formula to convert temperatures from Celsius to Fahrenheit and a different formula to convert temperatures from Fahrenheit to Celsius. Students should develop meaning for each equation based on the context of the problem.

Each student can develop both equations or you can split the class where part of your class will develop the formula for converting temperatures from Celsius to Fahrenheit, and the remaining students will develop a formula for converting Fahrenheit to Celsius. It is important that students explain the relationship between both formulas.

Mrs. Howell, your science teacher, overheard two of her students talking about how to convert temperatures from Celsius to Fahrenheit and vice versa. The students said they knew there was a formula, but they did not remember what it was. Mrs. Howell remarked to you that if they just knew about the freezing point and boiling point of water for each temperature scale, the formula could easily be “rediscovered.” Mrs. Howell has asked you to write a written explanation for how to find the formula, showing all your calculations. Mrs. Howell also wants you to include an explanation of each of the parts of the formula. (Don’t forget to include the formula for Celsius to Fahrenheit and the formula for Fahrenheit to Celsius.)

***Comments***

***Students will have to remember or research to find that the freezing point of water is 0 degrees Celsius and 32 degrees Fahrenheit. The boiling point of water is 100 degrees Celsius and 212 degrees Fahrenheit.***

***Solution***

***One method of finding this formula is to use (0, 32) and (100, 212) as two points on a line. To find the equation of the line only the slope is needed, since the y-intercept, (0, 32), is already given. Calculating the slope, , the students should simplify to . Substituting this for m in y = mx + b, the equation becomes . Because y represent the Fahrenheit temperature and x represents the Celsius temperature, the formula would be more appropriately written . F is the temperature expressed in degrees Fahrenheit. C is the temperature expressed in degrees Celsius. 32 is the y-intercept that means when it is 0 degrees Celsius it is 32 degrees Fahrenheit. The slope 9/5 shows that as the Celsius temperature increases or decreases five degrees that Fahrenheit will increase or decrease 9 degrees respectively.***

***Students could solve this equation for C to produce the other form expressing the relationship:***

***or***

***F is the temperature expressed in degrees Fahrenheit. C is the temperature expressed in degrees Celsius. is the y-intercept that means when it is 0 degrees Fahrenheit it is degrees Celsius. The slope 5/9 shows that as the Fahrenheit temperature increases or decreases nine degrees that Celsius will increase or decrease 5 degrees respectively.***

***They could also produce a graph of the corresponding temperatures***.

32

212

(°C, °F)

(100, 212)

Boiling point of water

(°C, °F)

(0, 32)

Freezing point of water

## Forget the Formula

**Mathematical Goals**

* Rearrange formulas to highlight a quantity of interest.
* Create equations in two variables to represent relationships.
* Understand how the change in one variable affects the other variable in a given situation.
* Write and graph an equation to represent a linear relationship.
* Extend the concepts used in solving numerical equations to rearranging formulas for a particular variable.

**Common Core State Standards**

**MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations*.*

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**8. Look for and express regularity in repeated reasoning.**

Mrs. Howell, your science teacher, overheard two of her students talking about how to convert temperatures from Celsius to Fahrenheit and vice versa. The students said they knew there was a formula, but they did not remember what it was. Mrs. Howell remarked to you that if they just knew about the freezing point and boiling point of water for each temperature scale, the formula could easily be “rediscovered.” Mrs. Howell has asked you to write a written explanation for how to find the formula, showing all your calculations. Mrs. Howell also wants you to include an explanation of each of the parts of the formula. (Don’t forget to include the formula for Celsius to Fahrenheit and the formula for Fahrenheit to Celsius.)

## Cara’s Candles Revisited

**Mathematical Goals**

* Determine whether a point is a solution to an equation.
* Determine whether a solution has meaning in a real-world context.
* Interpret whether the solution is viable from a given model.
* Write and graph equations and inequalities representing constraints in contextual situations.

**Common Core State Standards**

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

**Introduction**

In this task, students will create a table of values from a given scenario. After answering the question posed students will interpret whether the solution is viable or non-viable in modeling context. Students will also graph the equations to represent linear relationships.

**Materials**

* colored pencils
* graph paper
* graphing calculator (optional)

**Cara’s Candles Revisited**

Cara likes candles. She also likes mathematics and was thinking about using algebra to answer a question that she had about two of her candles. Her taller candle is 16 centimeters tall. Each hour it burns makes the candle lose 2.5 centimeters in height. Her short candle is 12 centimeters tall and loses 1.5 centimeters in height for each hour that it burns.

Cara started filling out the following table to help determine whether these two candles would ever reach the same height at the same time if allowed to burn the same length of time. Finish the table for Cara. Use the data in the table to determine what time the two candles will be at the same height.

Also, she wants to know what height the two candles would be at that time. If it is not possible, she wants to know why it could not happen and what would need to be true in order for them to be able to reach the same height. To help Cara understand what you are doing, justify your results. Explain your thinking using the table and create a graphical representation of the situation.

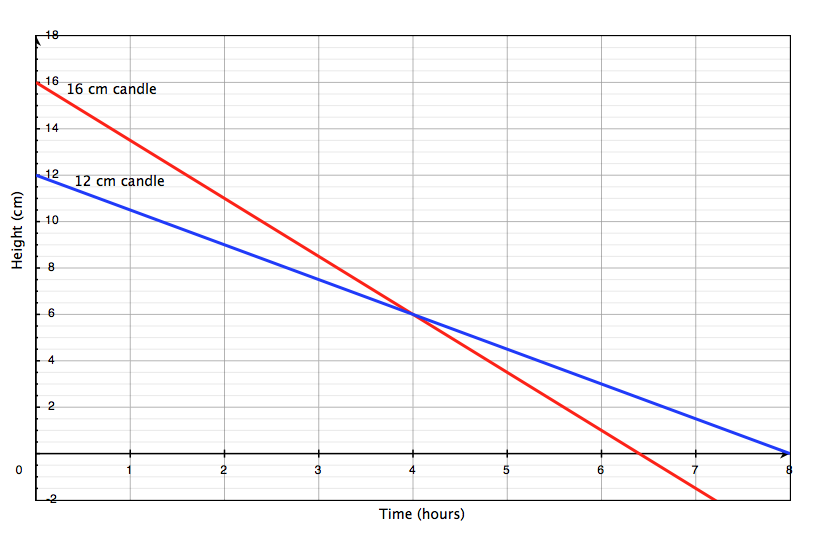
|  |  |  |
| --- | --- | --- |
| Time (hours) | 16 cm candle  height (cm) | 12 cm candle  height (cm) |
| 0 | 16 | 12 |
| 1 | 13.5 | 10.5 |
| 2 | ***11*** | ***9*** |
| 3 | ***8.5*** | ***7.5*** |
| 4 | ***6*** | ***6*** |
| 5 | ***3.5*** | ***4.5*** |
| 6 | ***1*** | ***3*** |
| 7 | ***-1.5*** | ***1.5*** |

***Solution***

***Students will use the following table to justify their solution. The candles will be the same height (6cm) in 4 hours.***

***Use the opportunity to bring out the concept of the natural restrictions.***

* ***For instance when x = 7 in the first function, the candle would have a negative height, which is impossible.***

******

Using the table, write an equation for the height of each candle in terms of the number of hours it has burned. Be sure to include any constraints for the equation.

***Taller Candle***

***y = -2.5x + 16; 0 < x < 6.4 hours***

***Shorter Candle***

***y - -1.5x + 12; 0 < x < 8 hours***

Cara has another candle that is 15 cm tall. How fast must it burn in order to also be 6 cm tall after 4 hours? Explain your thinking.

***The candle would need to lose 9 cm in four hours so it would have to burn at the rate of 2.25 cm per hour. The slope of its linear equation would be -2.25***

If Cara had a candle that burned 3 cm every hour, how tall would it need to be to also reach the same height as the other three candles after 4 hours? Explain your thinking.

***The candle would have burned 12 cm in 4 hours. Its initial height would have been 18 cm tall. The y-intercept of its linear equation would be 18.***

## Cara’s Candles Revisited

**Mathematical Goals**

* Determine whether a point is a solution to an equation.
* Determine whether a solution has meaning in a real-world context.
* Interpret whether the solution is viable from a given model.
* Write and graph equations and inequalities representing constraints in contextual situations.

**Common Core State Standards**

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

**Cara’s Candles Revisited**

Cara likes candles. She also likes mathematics and was thinking about using algebra to answer a question that she had about two of her candles. Her taller candle is 16 centimeters tall. Each hour it burns makes the candle lose 2.5 centimeters in height. Her short candle is 12 centimeters tall and loses 1.5 centimeters in height for each hour that it burns.

Cara started filling out the following table to help determine whether these two candles would ever reach the same height at the same time if allowed to burn the same length of time. Finish the table for Cara. Use the data in the table to determine what time the two candles will be at the same height.

Also, she wants to know what height the two candles would be at that time. If it is not possible, she wants to know why it could not happen and what would need to be true in order for them to be able to reach the same height. To help Cara understand what you are doing, justify your results. Explain your thinking using the table and create a graphical representation of the situation.

|  |  |  |
| --- | --- | --- |
| Time (hours) | 16 cm candle  height (cm) | 12 cm candle  height (cm) |
| 0 | 16 | 12 |
| 1 | 13.5 | 10.5 |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |

Using the table, write an equation for the height of each candle in terms of the number of hours it has burned. Be sure to include any constraints for the equation.

Cara has another candle that is 15 cm tall. How fast must it burn in order to also be 6 cm tall after 4 hours? Explain your thinking.

If Cara had a candle that burned 3 cm every hour, how tall would it need to be to also reach the same height as the other three candles after 4 hours? Explain your thinking.

## The Yo-Yo Problem

## 

Adapted from PBS Mathline: <http://www.pbs.org/teachers/mathline/lessonplans/pdf/hsmp/yoyo.pdf>

**Mathematical Goals**

* Explore linear patterns.
* Create one variable and two variable linear equations.
* Graph equations on coordinate axes with labels and scales.

**Common Core State Standards**

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions~~, ~~and simple rational and~~ exponential functions.

**MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MCC9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**4. Model with mathematics.**

**6. Attend to precision.**

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

**Introduction**

The lesson starts with the presentation of the yo-yo problem. Students then complete a hands-on activity involving a design created with pennies that allows them to explore a linear pattern and express that pattern in symbolic form.

**Materials**

* 31 pennies

Part 1: The Yo-Yo Problem

Andy wants to buy a very special yo-yo. He is hoping to be able to save enough money to buy it in time to take a class in which he would learn how to do many fancy tricks. The 5-ounce aluminum yo-yo costs $89.99 plus 6% sales tax. Andy has already saved $17.25, and he is earning $7.20 a week by doing odd jobs and chores. How many weeks will it take him to save enough money for the yo-yo?

1. How much sales tax will Andy have to pay? ***$5.40***
2. What will be the total cost of the yo-yo, including tax? ***$95.39***
3. Let *w* be the number of weeks that it will take Andy to save enough money to buy the yo-yo. Write an algebraic equation that will help you solve the problem.

***17.25 + 7.20w = 95.39***

1. Solve your equation for *w*, and check your answer. Be prepared to present your solution to the class. ***w = 10.8528 or approximately 11 weeks***

***Comments***

***Students need to be reminded when to approximate solutions in terms of the context of the problem.***



Part 2: The Penny Pattern

1. Create a pattern using pennies. Stage one of the pattern is shown next to the title above—one penny surrounded by six additional pennies. To create each additional stage of the design, place more pennies extending out from the six that surround the center penny. Continue making this design until you have used up all of your pennies. On the back of this sheet, sketch the first four stages of the pattern.
2. Using your penny pattern or the sketches of your penny pattern, create a table of values.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Stage Number, n | 1 | 2 | 3 | 4 | 5 |
| Number of Pennies Required | ***7*** | ***13*** | ***19*** | ***25*** | ***31*** |

1. How many pennies are needed to make stage 6, stage 7, and stage 8 of the penny pattern? How did you determine your answer? ***Stages 6, 7, 8 require 37, 43, and 49 pennies.***
2. Write an algebraic model that expresses the relationship between the stage number, *n*, and the number of pennies required to make that design, *p*. ***p = 1 + 6n***
3. Use your model to determine how many pennies are needed to make stage 80, stage 95, and stage 100 of the penny pattern. ***Stages 80, 95, and 100 require 481, 571, and 601***
4. If you use 127 pennies to make the penny pattern, how many pennies will be in each spoke coming out from the center penny?

***1 + 6n = 127***

***6n = 126***

***n = 21***

## *There will 21 pennies in each spoke coming out from the center penny.* The Yo-Yo Problem

## 

Adapted from PBS Mathline: <http://www.pbs.org/teachers/mathline/lessonplans/pdf/hsmp/yoyo.pdf>

**Mathematical Goals**

* Explore linear patterns.
* Create one variable and two variable linear equations.
* Graph equations on coordinate axes with labels and scales.

**Common Core State Standards**

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions~~, ~~and simple rational and~~ exponential functions.

**MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MCC9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**4. Model with mathematics.**

**6. Attend to precision.**

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

Part 1: The Yo-Yo Problem

Andy wants to buy a very special yo-yo. He is hoping to be able to save enough money to buy it in time to take a class in which he would learn how to do many fancy tricks. The 5-ounce aluminum yo-yo costs $89.99 plus 6% sales tax. Andy has already saved $17.25, and he is earning $7.20 a week by doing odd jobs and chores. How many weeks will it take him to save enough money for the yo-yo?

1. How much sales tax will Andy have to pay?
2. What will be the total cost of the yo-yo, including tax?
3. Let *w* be the number of weeks that it will take Andy to save enough money to buy the yo-yo. Write an algebraic equation that will help you solve the problem.
4. Solve your equation for *w*, and check your answer. Be prepared to present your solution to the class.

Part 2: The Penny Pattern

1. Create a pattern using pennies. Stage one of the pattern is shown above—one penny surrounded by six additional pennies. To create each additional stage of the design, place more pennies extending out from the six that surround the center penny. Continue making this design until you have used up all of your pennies. On the back of this sheet, sketch the first four stages of the pattern.
2. Using your penny pattern or the sketches of your penny pattern, create a table of values.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Stage Number, n | 1 | 2 | 3 | 4 | 5 |
| Number of Pennies Required |  |  |  |  |  |

1. How many pennies are needed to make stage 6, stage 7, and stage 8 of the penny pattern? How did you determine your answer?
2. Write an algebraic model that expresses the relationship between the stage number, *n*, and the number of pennies required to make that design, *p*.
3. Use your model to determine how many pennies are needed to make stage 80, stage 95, and stage 100 of the penny pattern.
4. If you use 127 pennies to make the penny pattern, how many pennies will be in each spoke coming out from the center penny?

Paper Folding

Adapted from PBS Mathline: <http://www.pbs.org/teachers/mathline/lessonplans/pdf/hsmp/rhinos.pdf>

**Mathematical Goals**

* Write and graph an equation to represent an exponential relationship.
* Model a data set using an equation.
* Choose the best form of an equation to model exponential functions.
* Use properties of exponents to solve and interpret the solution to exponential equations in context.
* Graph equations on coordinate axes with labels and scales.

**Common Core State Standards**

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions~~, ~~and simple rational and~~ exponential functions.

**MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MCC9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

**6. Attend to precision.**

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

**Introduction**

Students will use paper folding to model exponential functions. Students will collect data, create scatterplots, and determine algebraic models that represent their functions. Students begin this lesson by collecting data within their groups. They fold a sheet of paper and determine the area of the smallest region after each fold. Next they draw a scatterplot of their data and determine by hand an algebraic model. This investigation allows students to explore the patterns of exponential models in tables, graphs, and symbolic form.

**Materials**

• graphing calculator (optional)

• graph paper

***Solution***

**Part 1: Number of Sections**

|  |  |
| --- | --- |
| **Number of Folds** | **Number of Sections** |
| **0** | ***1*** |
| **1** | ***2*** |
| **2** | ***4*** |
| **3** | ***8*** |
| **4** | ***16*** |
| **5** | ***32*** |
| **6** | ***64*** |

Determine a mathematical model that represents this data by examining the patterns in the table.

***y = 2x***

***Comments***

***Students will need guidance in determining a mathematical model that represents the data. This is their first exposure to modeling exponential functions.***

What might be different if you tried this experiment with an 8.5 x 11” sheet of wax paper or tissue paper?

***The results would be exactly the same, but you would be able to make more folds and collect more data because the paper would be thinner.***

**Part 2: Area of Smallest Section**

|  |  |
| --- | --- |
| **Number of Folds** | **Area of Smallest Section** |
| **0** | 1 |
| **1** | ***½*** |
| **2** | ***¼*** |
| **3** | ***1/8*** |
| **4** | ***1/16*** |
| **5** | ***1/32*** |
| **6** | ***1/64*** |

Determine a mathematical model that represents this data by examining the patterns in the table.

**y = (1/2)x**

***Comments***

***Students will need guidance in determining a mathematical model that represents the data. This is their first exposure to modeling exponential functions.***

Paper Folding

Adapted from PBS Mathline: <http://www.pbs.org/teachers/mathline/lessonplans/pdf/hsmp/rhinos.pdf>

**Mathematical Goals**

* Write and graph an equation to represent an exponential relationship.
* Model a data set using an equation.
* Choose the best form of an equation to model exponential functions.
* Use properties of exponents to solve and interpret the solution to exponential equations in context.
* Graph equations on coordinate axes with labels and scales.

**Common Core State Standards**

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions~~, ~~and simple rational and~~ exponential functions.

**MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MCC9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

**6. Attend to precision.**

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

The Paper Folding Activity

**Part 1: Number of Sections**

|  |  |
| --- | --- |
| **Number of Folds** | **Number of Sections** |
| **0** |  |
| **1** |  |
| **2** |  |
| **3** |  |
| **4** |  |
| **5** |  |
| **6** |  |

1. Fold an 8.5 x 11” sheet of paper in half and determine the number of sections the paper has after you have made the fold.
2. Record this data in the table and continue in the same manner until it becomes too hard to fold the paper.
3. Make a scatter plot of your data in a separate sheet of graph paper.
4. Determine a mathematical model that represents this data by examining the patterns in the table.
5. What might be different if you tried this experiment with an 8.5 x 11” sheet of wax paper or tissue paper?

**Part 2: Area of Smallest Section**

|  |  |
| --- | --- |
| **Number of Folds** | **Area of Smallest Section** |
| **0** | 1 |
| **1** |  |
| **2** |  |
| **3** |  |
| **4** |  |
| **5** |  |
| **6** |  |

1. Fold an 8.5 x 11” sheet of paper in half and determine the area of the smallest section after you have made the fold.
2. Record this data in the table and continue in the same manner until it becomes too hard to fold the paper.
3. Make a scatter plot of your data on a separate sheet of graph paper.
4. Determine a mathematical model that represents this data by examining the patterns in the table.

## Culminating Task: Growing by Leaps and Bounds

**Mathematical Goals**

* Create one-variable exponential equations from contextual situations.
* Use properties of exponents to solve and interpret the solution to exponential equations in context.
* Write and graph an equation to represent an exponential relationship.
* Graph equations on coordinate axes with labels and scales.
* Use technology to explore exponential graphs.

**Common Core State Standards**

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions~~, ~~and simple rational and~~ exponential functions.

**MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations*.*

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MCC9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

**Introduction**

This task introduces students to exponential functions. At this point in their study, students have extended their understanding of exponents to include all integer values but have not yet discussed rational or real number exponents. In Part 1, students investigate a mathematical model of spreading a rumor in which the domain of the function is limited to a finite set of nonnegative integers. In Part 2, students learn the definition of an exponential function and see the model from Part 1 as an example of such a function. The emphasis in Part 2 is the pattern for the formula of an exponential function and an introduction to the shape of the graph. In Part 3, students work with the compound interest formula.

**Materials**

* Graph paper
* Graphing utility
* Optional: spreadsheet software

**Part 1: Meet Linda**

Linda’s lifelong dream has been to open her own business. After working, sacrificing, and saving, she finally has enough money to open up an ice cream business. The grand opening of her business is scheduled for the Friday of Memorial Day weekend. She would like to have a soft opening for her business on the Tuesday before. The soft opening should give her a good idea of any supply or personnel issues and give her time to correct them before the big official opening.

A soft opening means that the opening of the business is not officially announced; news of its opening is just spread by word of mouth (see, not all rumors are bad!). Linda needs a good idea of when she should begin the rumor in order for it to spread reasonably well before her soft opening. She has been told that about 10% of the people who know about an event will actually attend it. Based on this assumption, if she wants to have about 50 people visit her store on the Tuesday of the soft opening, she will need 500 people to know about it.

1. Linda plans to tell one person each day and will ask that person to tell one other person each day through the day of the opening, and so on. Assume that each new person who hears about the soft opening is also asked to tell one other person each day through the day of the opening and that each one starts the process of telling their friends on the day after he or she first hears. When should Linda begin telling others about the soft opening in order to have at least 500 people know about it by the day it occurs?

***Comments***

***With the table in item 2 below, it is likely that many students will organize their work in a similar way. Whether or not they use such a table, they will need to count up from the first day that Linda begins to spread the news to find out how many days it will take for the number of people who know to reach 500 and then count backwards to determine the day Linda should start. The Memorial Day reference in the problem gives a convenient way to express the answer.***

***Solutions***

***Linda should tell her first person about the soft opening on Monday two weeks before Memorial Day because:***

***1st day: Linda tells one other person – 2 people know.***

***2nd day: Each of the two people who know tell another person – 4 people know.***

***3rd day: Each of the four people who know tell another person – 8 people know.***

***4th day: 16 people know***

***5th day: 32 people know***

***6th day: 64 people know***

***7th day: 128 people know***

***8th day: 256 people know***

***9th day: 512 people know***

***The 9th day corresponds to the Tuesday of the soft opening. So, the 2nd day is the Tuesday one week before, and the 1st day is the Monday that is two weeks before Memorial Day.***

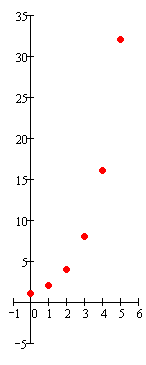
1. Let *x* represent the day number and let *y* be the number of people who know about the soft opening on day *x*. Consider the day before Linda told anyone to be Day 0, so that Linda is the only person who knows about the opening on Day 0. Day 1 is the first day that Linda told someone else about the opening.
   1. Complete the following table.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Day | 0 | 1 | 2 | 3 | 4 | 5 |
| Number of people who know | 1 | 2 | **4** | **8** | **16** | **32** |

***Comments***

***The table of values is limited to fewer than that needed to answer the question in item 1 so that the graph in part b below will show the y-intercept and the shape typical of an exponential function.***

***Solutions***

******

***The completed table is shown above.***

* 1. Graph the points from the table in part a.

***Comments***

***Since this is the students’ first experience with an exponential graph, it is recommended that students draw this graph by hand on graph paper.***

***Solutions***

***The graph is shown at the right.***

1. Write an equation that describes the relationship between *x* (day) and *y* (number of people who know) for the situation of spreading the news about the soft opening of Linda’s ice cream store.

***Comments***

***Students should easily see that the outputs of the function are powers of* 2 *and then note that the day number and the power of* 2 *are the same.***

***Solutions***

******

1. Does your equation describe the relationship between day and number who know about Linda’s ice cream store soft opening completely? Why or why not?

***Comments***

***The point of this question is that students realize that the domain is restricted in ways not implied by the equation. Since students have not yet studied a definition for non-integer exponents, they may believe that the equation makes sense only for integer exponents. However, they know about negative integer exponents and thus need to explicitly excluded these from the domain. They also need to exclude integers greater than 9 from the domain since Linda’s method of spreading the news of the soft opening stops on the day of the opening. If students state the correct inequalities but do not explicitly state that the exponents should be integers, teachers need to explain that this restriction must be included since other numbers can be exponents, although they will not study other exponents explicitly until a later course.***

***Solutions***

***No, the equation does not describe the relationship completely because the domain needs to be restricted to the integers* 0, 1, 2, . . . , 9, *and this information is not included in the equation.***

**Part 2: What if?**

The spread of a rumor or the spread of a disease can be modeled by a type of function known as exponential function; in particular, an exponential ***growth*** function. An **exponential function** has the form

,

where *a* is a non-zero real number and *b* is a positive real number other than 1. An exponential growth function has a value of b that is greater than 1.

In the case of Linda’s ice cream store, what values of *a* and *b* yield an exponential function to model the spread of the rumor of the soft store opening?

***Comments***

***For the rumor model, the coefficient a has a value of 1. However, other exponential functions in this task have coefficients other than 1, so students are introduced to the general definition of an exponential function from the beginning.***

***Solutions***

***a =* 1 *and b =* 2**

1. In this particular case, what is an appropriate domain for the exponential function? What range corresponds to this domain?

***Comments***

***Students are asked to specify the domain for this particular case. There is overlap with item 4 of part 1. The earlier question focused on whether all of the information is included in the equation. Here, the focus is explicitly to find the domain. Students may express the correct answer in a variety of ways. Two of these are shown in the solutions.***

***Solutions***

***The set of all nonnegative integers less than or equal to 9, or ***

1. In part 1, item 2, you drew a portion of the graph of this function. Does it make sense to

connect the dots on the graph? Why or why not?

***Comments***

***The question of whether to connect the dots was prominent in students’ early formal study of models in middle school. It reminds students to think of the meaning of points on the graph and to consider what values of the independent variable are meaningful in the situation. The description here is that the output is total number of people who know on a given day. Fractional parts of a day are not meaningful. Note: It would be impossible to draw an accurate model of this situation with a continuous time domain since we do not know when during the day each person who knows tells another person.***

***Solutions***

***No, it does not make sense to connect the dots. Connecting the dots would imply time passing continuously. We do not know when during the day people hear about the soft opening. We just have a count of the total number of people who know on each day.***

1. How would the graph change if Linda had told two people each day rather than one and had asked that each person also tell two other people each day?

***Comments***

***This question asks students to think in terms of function values or points on the graph, but they will have to think through the situation in a similar manner to the original. If students answer more generally here without being specific about new function values, then they will have more work to do in item 5 to find the new equation.***

***Solutions***

***The point* (0, 1) *would stay the same since on Day* 0 *Linda would still be the only person who knows about the opening. But for the other days, more people would know so the points for the other days would be higher.***

***In particular, on the first day,* 3 *people (Linda and the two people she tells) would know giving the point* (1, 3)*. On the second day,* 9 *people would know, because each of the* 3 *who know will tell* 2 *others giving a total of* 3 + 2(3) =9*. So the point for Day* 2 *is* (2, 9)*. We can continue in this way for the other points.***

1. How would the equation change if Linda had told two people each day rather than one and had asked that each person also tell two other people each day? What would be the values of *a* and *b* in this case?

***Comments***

***In addition to giving the formula, students must specify the values of a and b in order to reinforce the definition of exponential function.***

***Solutions***

***The equation would have a* 3 *as base for the exponent instead of a* 2*, that is, the equation would be . In this case, a =* 1 *and b =* 3.**

1. How long would it take for at least 500 people to find out about the opening if the rumor spread at this new rate?

***Comments***

***Students can use the graph or a table of values to determine the answer. If they draw a graph using a graphing utility, they should realize that the points of this function are only the integer valued points on the continuous graph shown. It is likely that most students will just count up to find the first power of* 3 *that is greater than 500.***

***Solutions***

***It would take 6 days for at least 500 people to find out.***

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ***Day number, x*** | **0** | **1** | **2** | **3** | **4** | **5** | **6** |
| ***No. people who know,*** | **1** | **3** | **9** | **27** | **81** | **243** | **729** |

**Part 3: The Beginning of a Business**

How in the world did Linda ever save enough to buy the franchise to an ice cream store? Her mom used to say, “That Linda, why she could squeeze a quarter out of a nickel!” The truth is that Linda learned early in life that patience with money is a good thing. When she was just about 9 years old, she asked her dad if she could put her money in the bank. He took her to the bank and she opened her very first savings account.

Each year until Linda was 16, she deposited her birthday money into her savings account. Her grandparents (both sets) and her parents each gave her money for her birthday that was equal to twice her age; so on her ninth birthday, she deposited $54 ($18 from each couple).

Linda’s bank paid her 3% interest, compounded quarterly. The bank calculated her interest using the following standard formula:



where *A* = final amount, *P* = principal amount, *r* = interest rate, *n* = number of times per year the interest is compounded, and *t* is the number of years the money is left in the account.

1. Verify the first entry in the following chart, and then complete the chart to calculate how much money Linda had on her 16th birthday. Do not round answers until the end of the computation, then give the final amount rounded to the nearest cent.

|  |  |  |  |
| --- | --- | --- | --- |
| Age | Birthday $ | Amt from previous year plus Birthday | Total at year end |
| 9 | 54 | 0 | 55.63831630 |
| 10 |  |  |  |
| 11 |  |  |  |
| 12 |  |  |  |
| **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** |

***Comments***

***Students will need exploration time to understand the compound interest formula. They may need to look up the term “principal amount” to understand that it refers to the amount deposited into the account. They will also need to realize that the interest rate r must be expressed in decimal form. If they enter numbers in their calculators following the formula exactly, they may need to be reminded about order of operations and that the calculator will not make the correct calculation unless the expression, nt, in the exponent is put in parentheses during calculation.***

***Some students may benefit from verifying the meaning of the compound interest formula by stepping through the compound interest calculation as four applications of simple interest using a rate of for each quarter for four quarters of one year as shown in the table below.***

|  |  |  |  |
| --- | --- | --- | --- |
| ***Quarter number*** | ***Amount invested at beginning of quarter*** | ***Amount of interest paid*** | ***Amount at end of quarter*** |
| **1** | **54** | **54(.0075) = 0.405** | **54.405** |
| **2** | **54.405** | **0.4080375** | **54.8130375** |
| **3** | **54.8130375** | **0.4110977813** | **55.22413528** |
| **4** | **55.22413528** | **0.4141810146** | **55.63831630** |

***Advanced students may benefit from seeing how the compound interest formula is developed using calculations similar to the above but using P for the amount of money, as shown below. One quarter is one-fourth of a year, so the number of quarters is always 4 times the number of years.***

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***No. of yrs*** | ***No. of qtrs*** | ***Amount invested at beginning of quarter*** | ***Amount of interest paid*** | ***Amount at end of quarter*** |
| **1/4** | **1** | ***P*** | ***P*(.0075)** | ***P*(1 + 0.0075)** |
| **1/2** | **2** | ***P*(1.0075)** | **[*P*(1.0075)](.0075)** | ***P*(1 + 0.0075)(1 + .0075) = *P*(1.0075)2** |
| **3/4** | **3** | ***P*(1.0075)2** | **[*P*(1.0075) 2](.0075)** | **[*P*(1.0075) 2](1 + .0075) = *P*(1.0075)3** |
| **1** | **4** | ***P*(1.0075)3** | **[*P*(1.0075) 3](.0075)** | **[*P*(1.0075) 3](1 + .0075) = *P*(1.0075)4** |
| **5/4** | **5** | ***P*(1.0075)4** | **[*P*(1.0075) 4](.0075)** | **[*P*(1.0075) 4](1 + .0075) = *P*(1.0075)5** |
| **3/2** | **6** | ***P*(1.0075)5** | **[*P*(1.0075) 5](.0075)** | **[*P*(1.0075) 5](1 + .0075) = *P*(1.0075)6** |

***Solutions***

***For her deposit at age* 9*, P =* 54*, r =* 0.03*, n =* 4*, t =* 4*.***

******

***, as in the chart***

|  |  |  |  |
| --- | --- | --- | --- |
| Age | $ received on this Birthday | Amt from previous year plus Birthday | Total at year end |
| 9 | 54 | 0 | 55.63832 |
| 10 | **60** | **115.63832** | **115.63832(1.0075)4 =119.14669** |
| 11 | **66** | **185.14669** | **185.14669(1.0075)4 = 190.76389** |
| 12 | **72** | **262.76389** | **262.76389(1.0075)4 = 270.73593** |
| **13** | **78** | **348.73593** | **348.73593(1.0075)4 = 359.31630** |
| **14** | **84** | **443.31630** | **443.31630(1.0075)4 = 456.76616** |
| **15** | **90** | **546.76616** | **546.76616(1.0075)4 = 563.35460** |

***On the day before her 16th birthday, a year after her 15th, Linda had $*563.35.**

1. On her 16th birthday, the budding entrepreneur asked her parents if she could invest in the stock market. She studied the newspaper, talked to her economics teacher, researched a few companies and finally settled on the stock she wanted. She invested all of her money in the stock and promptly forgot about it. When she graduated from college on her 22nd birthday, she received a statement from her stocks and realized that her stock had appreciated an average of 10% per year. How much was her stock worth on her 22nd birthday?

***Comments***

***The challenge for students here is realizing that the information that Linda’s stock had appreciated an average of 10% per year means that the money grew as if it were invested at 10% compounded annually for the 6 years.***

***Solutions***

***Linda’s stock was worth $998.01 by application of the compound interest formula with P =* 563.35, *r* = 0.10, *n* = 1, *and t* = 6:**

******

1. When Linda graduated from college, she received an academic award that carried a $500 cash award. On her 22nd birthday, she used the money to purchase additional stock. She started her first job immediately after graduation and decided to save $50 each month. On her 23rd birthday she used the $600 (total of her monthly amount) savings to purchase new stock. Each year thereafter she increased her the total of her savings by $100 and, on her birthday each year, used her savings to purchase additional stock. Linda continued to learn about stocks and managed her accounts carefully. On her 35th birthday she looked back and saw that her stock had appreciated at 11% during the first year after college and that the rate of appreciation increased by 0.25% each year thereafter. At age 34, she cashed in enough stock to make a down payment on a bank loan to purchase her business. What was her stock worth on her 34th birthday? Use a table like the one below to organize your calculations.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Age | Amt from previous year | Amt Linda added from savings that year | Amount invested for the year | Interest rate for the year | Amt at year end |
| 22 | 998.01 | 500 | 1498.01 | 11.00% | 1662.79 |
| 23 | 1662.79 | 600 |  | 11.25% |  |
| 24 |  | 700 |  | 11.50% |  |
| 25 |  | 800 |  | 11.75% |  |
| **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** |

***Comments***

***This item brings closure to this part of the learning task. The calculations are simple applications of the compound interest formula. If students have access to a spreadsheet program, having them set up the spreadsheet formulas is a possible extension of this activity.***

***Solutions***

***At age 34, Linda’s stock was worth* $30,133.63.**

***The completed table is given below as an embedded Excel file.***



***The same spreadsheet with formulas turned on is pasted in below.***

******

## Culminating Task: Growing by Leaps and Bounds

**Mathematical Goals**

* Create one-variable exponential equations from contextual situations.
* Use properties of exponents to solve and interpret the solution to exponential equations in context.
* Write and graph an equation to represent an exponential relationship.
* Graph equations on coordinate axes with labels and scales.
* Use technology to explore exponential graphs.

**Common Core State Standards**

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions~~, ~~and simple rational and~~ exponential functions.

**MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations*.*

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MCC9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

**Part 1: Meet Linda**

Linda’s lifelong dream has been to open her own business. After working, sacrificing, and saving, she finally has enough money to open up an ice cream business. The grand opening of her business is scheduled for the Friday of Memorial Day weekend. She would like to have a soft opening for her business on the Tuesday before. The soft opening should give her a good idea of any supply or personnel issues and give her time to correct them before the big official opening.

A soft opening means that the opening of the business is not officially announced; news of its opening is just spread by word of mouth (see, not all rumors are bad!). Linda needs a good idea of when she should begin the rumor in order for it to spread reasonably well before her soft opening. She has been told that about 10% of the people who know about an event will actually attend it. Based on this assumption, if she wants to have about 50 people visit her store on the Tuesday of the soft opening, she will need 500 people to know about it.

1. Linda plans to tell one person each day and will ask that person to tell one other person each day through the day of the opening, and so on. Assume that each new person who hears about the soft opening is also asked to tell one other person each day through the day of the opening and that each one starts the process of telling their friends on the day after he or she first hears. When should Linda begin telling others about the soft opening in order to have at least 500 people know about it by the day it occurs?
2. Let *x* represent the day number and let *y* be the number of people who know about the soft opening on day *x*. Consider the day before Linda told anyone to be Day 0, so that Linda is the only person who knows about the opening on Day 0. Day 1 is the first day that Linda told someone else about the opening.
   1. Complete the following table.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Day | 0 | 1 | 2 | 3 | 4 | 5 |
| Number of people who know | 1 | 2 |  |  |  |  |

* 1. Graph the points from the table in part a.

1. Write an equation that describes the relationship between *x* (day) and *y* (number of people who know) for the situation of spreading the news about the soft opening of Linda’s ice cream store.
2. Does your equation describe the relationship between day and number who know about Linda’s ice cream store soft opening completely? Why or why not?

**Part 2: What if?**

The spread of a rumor or the spread of a disease can be modeled by a type of function known as exponential function; in particular, an exponential ***growth*** function. An **exponential function** has the form

,

where *a* is a non-zero real number and *b* is a positive real number other than 1. An exponential growth function has a value of b that is greater than 1.

1. In the case of Linda’s ice cream store, what values of *a* and *b* yield an exponential function to model the spread of the rumor of the soft store opening?
2. In this particular case, what is an appropriate domain for the exponential function? What range corresponds to this domain?
3. In part 1, item 2, you drew a portion of the graph of this function. Does it make sense to connect the dots on the graph? Why or why not?
4. How would the graph change if Linda had told two people each day rather than one and had asked that each person also tell two other people each day?
5. How would the equation change if Linda had told two people each day rather than one and had asked that each person also tell two other people each day? What would be the values of *a* and *b* in this case?
6. How long would it take for at least 500 people to find out about the opening if the rumor spread at this new rate?

**Part 3: The Beginning of a Business**

How in the world did Linda ever save enough to buy the franchise to an ice cream store? Her mom used to say, “That Linda, why she could squeeze a quarter out of a nickel!” The truth is that Linda learned early in life that patience with money is a good thing. When she was just about 9 years old, she asked her dad if she could put her money in the bank. He took her to the bank and she opened her very first savings account.

Each year until Linda was 16, she deposited her birthday money into her savings account. Her grandparents (both sets) and her parents each gave her money for her birthday that was equal to twice her age; so on her ninth birthday, she deposited $54 ($18 from each couple).

Linda’s bank paid her 3% interest, compounded quarterly. The bank calculated her interest using the following standard formula:



where *A* = final amount, *P* = principal amount, *r* = interest rate, *n* = number of times per year the interest is compounded, and *t* is the number of years the money is left in the account.

1. Verify the first entry in the following chart, and then complete the chart to calculate how much money Linda had on her 16th birthday. Do not round answers until the end of the computation, then give the final amount rounded to the nearest cent.

|  |  |  |  |
| --- | --- | --- | --- |
| Age | Birthday $ | Amt from previous year plus Birthday | Total at year end |
| 9 | 54 | 0 | 55.63831630 |
| 10 |  |  |  |
| 11 |  |  |  |
| 12 |  |  |  |
| **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** |

1. On her 16th birthday, the budding entrepreneur asked her parents if she could invest in the stock market. She studied the newspaper, talked to her economics teacher, researched a few companies and finally settled on the stock she wanted. She invested all of her money in the stock and promptly forgot about it. When she graduated from college on her 22nd birthday, she received a statement from her stocks and realized that her stock had appreciated an average of 10% per year. How much was her stock worth on her 22nd birthday?
2. When Linda graduated from college, she received an academic award that carried a $500 cash award. On her 22nd birthday, she used the money to purchase additional stock. She started her first job immediately after graduation and decided to save $50 each month. On her 23rd birthday she used the $600 (total of her monthly amount) savings to purchase new stock. Each year thereafter she increased her total of her savings by $100 and, on her birthday each year, used her savings to purchase additional stock. Linda continued to learn about stocks and managed her accounts carefully. On her 35th birthday she looked back and saw that her stock had appreciated at 11% during the first year after college and that the rate of appreciation increased by 0.25% each year thereafter. At age 34, she cashed in enough stock to make a down payment on a bank loan to purchase her business. What was her stock worth on her 34th birthday? Use a table like the one below to organize your calculations.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Age | Amt from previous year | Amt Linda added from savings that year | Amount invested for the year | Interest rate for the year | Amt at year end |
| 22 | 998.01 | 500 | 1498.01 | 11.00% | 1662.79 |
| 23 | 1662.79 | 600 |  | 11.25% |  |
| 24 |  | 700 |  | 11.50% |  |
| 25 |  | 800 |  | 11.75% |  |
| **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** |